

Chapter 7. Indices (Exponents)

Exercise 7(A)

Solution 1:

(i)

$$\begin{aligned}3^3 \times (243)^{-\frac{2}{3}} \times 9^{-\frac{1}{3}} &= 3^3 \times (3 \times 3 \times 3 \times 3 \times 3)^{-\frac{2}{3}} \times (3 \times 3)^{-\frac{1}{3}} \\&= 3^3 \times (3^5)^{-\frac{2}{3}} \times (3^2)^{-\frac{1}{3}} \\&= 3^3 \times 3^{\left(-\frac{10}{3}\right)} \times 3^{-\frac{2}{3}} \quad [(a^m)^n = a^{mn}] \\&= 3^{3-\frac{10}{3}-\frac{2}{3}} \quad [a^m \times a^n \times a^p = a^{m+n+p}] \\&= 3^{\frac{9-10-2}{3}} \\&= 3^{\frac{9-12}{3}} \\&= 3^{-\frac{3}{3}} \\&= 3^{-1} \\&= \frac{1}{3}\end{aligned}$$

(ii)

$$\begin{aligned}5^{-4} \times (125)^{\frac{5}{3}} \div (25)^{-\frac{1}{2}} &= 5^{-4} \times (5 \times 5 \times 5)^{\frac{5}{3}} \div (5 \times 5)^{-\frac{1}{2}} \\&= 5^{-4} \times (5^3)^{\frac{5}{3}} \div (5^2)^{-\frac{1}{2}} \\&= 5^{-4} \times \left(5^{3 \times \frac{5}{3}}\right) \div \left(5^{2 \times \left(-\frac{1}{2}\right)}\right) \\&= \frac{5^{-4} \times 5^5}{5^{-1}} \\&= \frac{5^{5-4}}{5^{-1}} \\&= \frac{5^1}{5^{-1}} \\&= 5^{1-(-1)} \\&= 5^2 \\&= 5 \times 5 \\&= 25\end{aligned}$$

(iii)

$$\begin{aligned}\left(\frac{27}{125}\right)^{\frac{2}{3}} \times \left(\frac{9}{25}\right)^{-\frac{3}{2}} &= \left(\frac{3 \times 3 \times 3}{5 \times 5 \times 5}\right)^{\frac{2}{3}} \times \left(\frac{3 \times 3}{5 \times 5}\right)^{-\frac{3}{2}} \\&= \left[\left(\frac{3}{5}\right)^3\right]^{\frac{2}{3}} \times \left[\left(\frac{3}{5}\right)^2\right]^{-\frac{3}{2}} \\&= \left(\frac{3}{5}\right)^{3 \times \frac{2}{3}} \times \left(\frac{3}{5}\right)^{2 \times \left(-\frac{3}{2}\right)} \\&= \left(\frac{3}{5}\right)^2 \times \left(\frac{3}{5}\right)^{-3} \\&= \left(\frac{3}{5}\right)^{2-3} \\&= \left(\frac{3}{5}\right)^{-1} \\&= \frac{1}{\frac{3}{5}} \\&= \frac{5}{3}\end{aligned}$$

(iv)

$$\begin{aligned}7^0 \times (25)^{-\frac{3}{2}} - 5^{-3} &= 7^0 \times (5 \times 5)^{-\frac{3}{2}} - 5^{-3} \\&= 7^0 \times (5^2)^{-\frac{3}{2}} - \frac{1}{5^3} \\&= 7^0 \times 5^{2 \times \left(-\frac{3}{2}\right)} - \frac{1}{5^3} \\&= 7^0 \times 5^{-3} - \frac{1}{5^3} \\&= 1 \times 5^{-3} - \frac{1}{5^3} \\&= \frac{1}{5^3} - \frac{1}{5^3} \\&= \frac{1-1}{5 \times 5 \times 5} \\&= \frac{0}{125} \\&= 0\end{aligned}$$

(v)

$$\begin{aligned}& \left(\frac{16}{81}\right)^{-\frac{3}{4}} \times \left(\frac{49}{9}\right)^{\frac{3}{2}} \div \left(\frac{343}{216}\right)^{\frac{2}{3}} \\&= \left(\frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}\right)^{-\frac{3}{4}} \times \left(\frac{7 \times 7}{3 \times 3}\right)^{\frac{3}{2}} \div \left(\frac{7 \times 7 \times 7}{6 \times 6 \times 6}\right)^{\frac{2}{3}} \\&= \left[\left(\frac{2}{3}\right)^4\right]^{-\frac{3}{4}} \times \left[\left(\frac{7}{3}\right)^2\right]^{\frac{3}{2}} \div \left[\left(\frac{7}{6}\right)^3\right]^{\frac{2}{3}} \\&= \left(\frac{2}{3}\right)^{4 \times \left(-\frac{3}{4}\right)} \times \left(\frac{7}{3}\right)^{2 \times \frac{3}{2}} \div \left(\frac{7}{6}\right)^{3 \times \frac{2}{3}} \\&= \left(\frac{2}{3}\right)^{-3} \times \left(\frac{7}{3}\right)^3 \div \left(\frac{7}{6}\right)^2 \\&= \frac{1}{\left(\frac{2}{3}\right)^3} \times \left(\frac{7}{3}\right)^3 \times \frac{1}{\left(\frac{7}{6}\right)^2} \\&= \frac{1}{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}} \times \frac{7}{3} \times \frac{7}{3} \times \frac{7}{3} \times \frac{1}{\frac{7}{6} \times \frac{7}{6}} \\&= \frac{1 \times 3 \times 3 \times 3}{2 \times 2 \times 2} \times \frac{7}{3} \times \frac{7}{3} \times \frac{7}{3} \times \frac{1 \times 6 \times 6}{7 \times 7} \\&= \frac{7 \times 3 \times 3}{2} \\&= \frac{63}{2} \\&= 31.5\end{aligned}$$

Solution 2:

(i)

$$\begin{aligned}\left(8x^3 \div 125y^3\right)^{\frac{2}{3}} &= \left(\frac{8x^3}{125y^3}\right)^{\frac{2}{3}} \\&= \left(\frac{2x \times 2x \times 2x}{5y \times 5y \times 5y}\right)^{\frac{2}{3}} \\&= \left[\left(\frac{2x}{5y}\right)^3\right]^{\frac{2}{3}} \\&= \left(\frac{2x}{5y}\right)^{3 \times \frac{2}{3}} \\&= \left(\frac{2x}{5y}\right)^2 \\&= \frac{2x}{5y} \times \frac{2x}{5y} \\&= \frac{4x^2}{25y^2}\end{aligned}$$

(ii)

$$\begin{aligned}(a+b)^{-1} \cdot (a^{-1} + b^{-1}) &= \frac{1}{(a+b)} \times \left(\frac{1}{a} + \frac{1}{b} \right) \\&= \frac{1}{(a+b)} \times \left(\frac{b+a}{ab} \right) \\&= \frac{1}{(a+b)} \times \frac{(a+b)}{ab} \\&= \frac{1}{ab}\end{aligned}$$

(iii)

$$\begin{aligned}\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2} &= \frac{5^{n+1} \times 5^2 - 6 \times 5^{n+1}}{9 \times 5^n - 5^n \times 2^2} \\&= \frac{5^{n+1} \times (5^2 - 6)}{5^n \times (9 - 4)} \\&= \frac{5^n \times 5^1 \times (25 - 6)}{5^n \times (9 - 4)} \\&= \frac{5^1 \times 19}{5} \\&= 19\end{aligned}$$

(iv)

$$\begin{aligned}(3x^2)^{-3} \times (x^9)^{\frac{2}{3}} &= \frac{1}{(3x^2)^3} \times x^{9 \times \frac{2}{3}} \\&= \frac{1}{3^3 \times 2^3} \times x^6 \\&= \frac{1}{27 \times 8} \times x^6 \\&= \frac{1}{27}\end{aligned}$$

Solution 3:

(i)

$$\begin{aligned}
 \sqrt{\frac{1}{4}} + (0.01)^{-\frac{1}{2}} - (27)^{\frac{2}{3}} &= \sqrt{\frac{1}{2} \times \frac{1}{2}} + (0.1 \times 0.1)^{-\frac{1}{2}} - (3 \times 3 \times 3)^{\frac{2}{3}} \\
 &= \frac{1}{2} + \left[(0.1)^2\right]^{\frac{1}{2}} - (3^3)^{\frac{2}{3}} \\
 &= \frac{1}{2} + (0.1)^{2 \times \left(-\frac{1}{2}\right)} - 3^{3 \times \frac{2}{3}} \\
 &= \frac{1}{2} + (0.1)^{-1} - 3^2 \\
 &= \frac{1}{2} + \frac{1}{0.1} - 9 \\
 &= \frac{1}{2} + \frac{10}{1} - 9 \\
 &= \frac{1 + 20 - 18}{2} \\
 &= \frac{3}{2}
 \end{aligned}$$

(ii)

$$\begin{aligned}
 \left(\frac{27}{8}\right)^{\frac{2}{3}} - \left(\frac{1}{4}\right)^{-2} + 5^0 &= \left(\frac{3 \times 3 \times 3}{2 \times 2 \times 2}\right)^{\frac{2}{3}} - \left(\frac{1 \times 1}{2 \times 2}\right)^{-2} + 5^0 \\
 &= \left[\left(\frac{3}{2}\right)^3\right]^{\frac{2}{3}} - \left[\left(\frac{1}{2}\right)^2\right]^{-2} + 1 \\
 &= \left(\frac{3}{2}\right)^{3 \times \frac{2}{3}} - \left(\frac{1}{2}\right)^{2 \times (-2)} + 1 \\
 &= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^{-4} + 1 \\
 &= \frac{3}{2} \times \frac{3}{2} - \frac{1}{\left(\frac{1}{2}\right)^4} + 1 \\
 &= \frac{9}{4} - \frac{1}{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}} + 1 \\
 &= \frac{9}{4} - \frac{1}{\frac{1}{16}} + 1 \\
 &= \frac{9}{4} - 16 + 1 \\
 &= \frac{9 - 64 + 4}{4} \\
 &= \underline{\underline{-51}}
 \end{aligned}$$

Solution 4:

(i)

$$\begin{aligned}\left(\frac{3^{-4}}{2^{-8}}\right)^{\frac{1}{4}} &= \left(\frac{2^8}{3^4}\right)^{\frac{1}{4}} \\&= \frac{(2^8)^{\frac{1}{4}}}{(3^4)^{\frac{1}{4}}} \\&= \frac{2^{8 \times \frac{1}{4}}}{3^{4 \times \frac{1}{4}}} \\&= \frac{2^2}{3} \\&= \frac{4}{3}\end{aligned}$$

(ii)

$$\begin{aligned}\left(\frac{27^{-3}}{9^{-3}}\right)^{\frac{1}{5}} &= \left(\frac{9^3}{27^3}\right)^{\frac{1}{5}} \\&= \left(\frac{(3^2)^3}{(3^3)^3}\right)^{\frac{1}{5}} \\&= \left[\left(\frac{3^2}{3^3}\right)^3\right]^{\frac{1}{5}} \\&= \left[\left(\frac{1}{3}\right)^3\right]^{\frac{1}{5}} \\&= \left(\frac{1}{3}\right)^{3 \times \frac{1}{5}} \\&= \frac{1}{3^{\frac{3}{5}}}\end{aligned}$$

(iii)

$$\begin{aligned}(32)^{-\frac{2}{5}} \div (125)^{-\frac{2}{3}} &= \frac{(32)^{-\frac{2}{5}}}{(125)^{-\frac{2}{3}}} \\&= \frac{(125)^{\frac{2}{3}}}{(32)^{\frac{2}{5}}} \\&= \frac{(5 \times 5 \times 5)^{\frac{2}{3}}}{(2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}}} \\&= \frac{(5^3)^{\frac{2}{3}}}{(2^5)^{\frac{2}{5}}} \\&= \frac{5^2}{2^2} \\&= \frac{25}{4} \\&= 6\frac{1}{4}\end{aligned}$$

(iv)

$$\left[1 - \left\{1 - (1 - n)^{-1}\right\}^{-1}\right]^1 = \frac{1}{\left[1 - \left\{1 - (1 - n)^{-1}\right\}^{-1}\right]^{+1}}$$



$$\begin{aligned}
&= \frac{1}{1 - \frac{1}{1 - (1-n)^{-1}}} \\
&= \frac{1}{1 - \frac{1}{1 - \frac{1}{(1-n)}}} \\
&= \frac{1}{1 - \frac{1}{\frac{1(1-n) - 1}{(1-n)}}} \\
&= \frac{1}{1 - \frac{1}{\frac{1-n-1}{(1-n)}}} \\
&= \frac{1}{1 - \frac{1}{\frac{-n}{(1-n)}}} \\
&= \frac{1}{1 - \frac{(1-n)}{-n}} \\
&= \frac{1}{1 + \frac{(1-n)}{n}} \\
&= \frac{1}{\frac{n + (1-n)}{n}} \\
&= \frac{1}{\frac{n + 1 - n}{n}} \\
&= \frac{n}{1} \\
&= n
\end{aligned}$$

Solution 5:

$$2160 = 2^a \times 3^b \times 5^c$$

$$\Rightarrow 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^a \times 3^b \times 5^c$$

$$\Rightarrow 2^4 \times 3^3 \times 5^1 = 2^a \times 3^b \times 5^c$$

$$\Rightarrow 2^a \times 3^b \times 5^c = 2^4 \times 3^3 \times 5^1$$

Comparing powers of 2, 3 and 5 on the both sides of equation, we have

$$a=4; b=3 \text{ and } c=1$$

$$\text{Hence value of } 3^a \times 2^{-b} \times 5^{-c} = 3^4 \times 2^{-3} \times 5^{-1}$$

$$= 3 \times 3 \times 3 \times 3 \times \frac{1}{2^3} \times \frac{1}{5}$$

$$= 81 \times \frac{1}{2 \times 2 \times 2} \times \frac{1}{5}$$

$$= 81 \times \frac{1}{8} \times \frac{1}{5}$$

$$= \frac{81}{40}$$

$$= 2\frac{1}{40}$$

Solution 6:

$$1960 = 2^a \times 5^b \times 7^c$$

$$\Rightarrow 2 \times 2 \times 2 \times 5 \times 7 \times 7 = 2^a \times 5^b \times 7^c$$

$$\Rightarrow 2^3 \times 5^1 \times 7^2 = 2^a \times 5^b \times 7^c$$

$$\Rightarrow 2^a \times 5^b \times 7^c = 2^3 \times 5^1 \times 7^2$$

Comparing powers of 2, 5 and 7 on the both sides of equation, we have

$$a=3; b=1 \text{ and } c=2$$

$$\text{Hence value of } 2^{-a} \times 7^b \times 5^{-c} = 2^{-3} \times 7^1 \times 5^{-2}$$

$$= \frac{1}{2^3} \times 7 \times \frac{1}{5^2}$$

$$= \frac{1}{8} \times 7 \times \frac{1}{5 \times 5}$$

$$= \frac{7}{200}$$

Solution 7:

(i)

$$\begin{aligned}
 \frac{8^{3a} \times 2^5 \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}} &= \frac{(2^3)^{3a} \times 2^5 \times 2^{2a}}{2^2 \times 2^{11a} \times 2^{-2a}} \\
 &= \frac{2^{3 \times 3a} \times 2^5 \times 2^{2a}}{2^2 \times 2^{11a} \times 2^{-2a}} \\
 &= \frac{2^{9a} \times 2^5 \times 2^{2a}}{2^2 \times 2^{11a} \times 2^{-2a}} \\
 &= 2^{9a+5+2a-2-11a+2a} \\
 &= 2^{2a+3}
 \end{aligned}$$

(ii)

$$\begin{aligned}
 \frac{3 \times 27^{n+1} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 27^n} &= \frac{3 \times (3 \times 3 \times 3)^{n+1} + 3 \times 3 \times 3^{3n-1}}{2 \times 2 \times 2 \times 3^{3n} - 5 \times (3 \times 3 \times 3)^n} \\
 &= \frac{3 \times (3^3)^{n+1} + 3^2 \times 3^{3n-1}}{2^3 \times 3^{3n} - 5 \times (3^3)^n} \\
 &= \frac{3 \times 3^{3n+3} + 3^{3n+1}}{2^3 \times (3^3)^n - 5 \times (3^3)^n} \\
 &= \frac{3^{3n+3+1} + 3^{3n+1}}{2^3 \times (3^3)^n - 5 \times (3^3)^n} \\
 &= \frac{3^{3n+4} + 3^{3n+1}}{2^3 \times (3^3)^n - 5 \times (3^3)^n} \\
 &= \frac{3^{3n} \times 3^4 + 3^{3n} \times 3^1}{2^3 \times (3^3)^n - 5 \times (3^3)^n} \\
 &= \frac{3^{3n} (3^4 + 3^1)}{(3^3)^n (8 - 5)} \\
 &= \frac{3^{3n} (3^4 + 3^1)}{3^{3n} \times 3} \\
 &= \frac{3 \times 3 \times 3 \times 3 + 3}{3} \\
 &= \frac{81 + 3}{3} \\
 &= \frac{84}{3} \\
 &= 28
 \end{aligned}$$

Solution 8:

$$\begin{aligned}
& \left(\frac{a^m}{a^{-n}} \right)^{m-n} \times \left(\frac{a^n}{a^{-l}} \right)^{n-l} \times \left(\frac{a^l}{a^{-m}} \right)^{l-m} \\
&= \left(a^m \times a^n \right)^{m-n} \times \left(a^n \times a^l \right)^{n-l} \times \left(a^l \times a^m \right)^{l-m} \\
&= \left(a^{m+n} \right)^{m-n} \times \left(a^{n+l} \right)^{n-l} \times \left(a^{l+m} \right)^{l-m} \\
&= a^{m^2-n^2} \times a^{n^2-l^2} \times a^{l^2-m^2} \\
&= a^{m^2-n^2+n^2-l^2+l^2-m^2} \\
&= a^0 \\
&= 1
\end{aligned}$$

Solution 9:

$$a = x^{m+n} \cdot x^l$$

$$b = x^{n+l} \cdot x^m$$

$$c = x^{l+m} \cdot x^n$$

LHS

$$a^{m-n} \cdot b^{n-l} \cdot c^{l-m}$$

$$= (x^{m+n} \cdot x^l)^{m-n} \cdot (x^{n+l} \cdot x^m)^{n-l} \cdot (x^{l+m} \cdot x^n)^{l-m} \text{ [Substituting a,b,c in LHS]}$$

$$= x^{(m+n)(m-n)} \cdot x^{l(m-n)} \cdot x^{(n+l)(n-l)} \cdot x^{m(n-l)} \cdot x^{(l+m)(l-m)} \cdot x^{n(l-m)}$$

$$= x^{m^2-n^2+ml-nl+n^2-l^2+mn-nl+l^2-m^2+nl-mn}$$

$$= x^0$$

$$= 1 = \text{RHS}$$

Solution 10:

(i)

$$\begin{aligned}
& \left(\frac{x^a}{x^b} \right)^{a^2+ab+b^2} \times \left(\frac{x^b}{x^c} \right)^{b^2+bc+c^2} \times \left(\frac{x^c}{x^a} \right)^{c^2+ca+a^2} \\
&= \left(x^{a-b} \right)^{a^2+ab+b^2} \times \left(x^{b-c} \right)^{b^2+bc+c^2} \times \left(x^{c-a} \right)^{c^2+ca+a^2} \\
&= x^{a^3-b^3} \times x^{b^3-c^3} \times x^{c^3-a^3} \\
&= x^{a^3-b^3+b^3-c^3+c^3-a^3} \\
&= x^0 \\
&= 1
\end{aligned}$$

(ii)

$$\begin{aligned}
& \left(\frac{x^a}{x^{-b}} \right)^{a^2-ab+b^2} \times \left(\frac{x^b}{x^{-c}} \right)^{b^2-bc+c^2} \times \left(\frac{x^c}{x^{-a}} \right)^{c^2-ca+a^2} \\
&= \left(x^{a+b} \right)^{a^2-ab+b^2} \times \left(x^{b+c} \right)^{b^2-bc+c^2} \times \left(x^{c+a} \right)^{c^2-ca+a^2} \\
&= x^{a^3+b^3} \times x^{b^3+c^3} \times x^{c^3+a^3} \\
&= x^{a^3+b^3+b^3+c^3+c^3+a^3} \\
&= x^{(a^3+b^3+b^3+c^3+c^3+a^3)} \\
&= x^{2(a^3+b^3+c^3)}
\end{aligned}$$

Exercise 7(B)

Solution 1:

(i)

$$2^{2x+1} = 8$$

$$\Rightarrow 2^{2x+1} = 2^3$$

We know that if bases are equal, the powers are equal

$$\Rightarrow 2x+1=3$$

$$\Rightarrow 2x=3-1$$

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = \frac{2}{2}$$

$$\Rightarrow x = 1$$

(ii)

$$2^{5x-1} = 4 \times 2^{3x+1}$$

$$\Rightarrow 2^{5x-1} = 2^2 \times 2^{3x+1}$$

$$\Rightarrow 2^{5x-1} = 2^{3x+1+2}$$

$$\Rightarrow 2^{5x-1} = 2^{3x+3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow 5x-1=3x+3$$

$$\Rightarrow 5x-3x=3+1$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = \frac{4}{2}$$

$$\Rightarrow x = 2$$

(iii)

$$3^{4x+1} = 27^{(x+1)}$$

$$\Rightarrow 3^{4x+1} = (3^3)^{x+1}$$

$$\Rightarrow 3^{4x+1} = 3^{3x+3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow 4x+1=3x+3$$

$$\Rightarrow 4x-3x=3-1$$

$$\Rightarrow x = 2$$

(iv)

$$49^{x+4} = 7^2 (343)^{(x+1)}$$

$$\Rightarrow (7 \times 7)^{x+4} = 7^2 (7 \times 7 \times 7)^{(x+1)}$$

$$\Rightarrow (7^2)^{x+4} = 7^2 (7^3)^{(x+1)}$$

$$\Rightarrow 7^{2x+8} = 7^2 \times 7^{3x+3}$$

$$\Rightarrow 7^{2x+8} = 7^{3x+3+2}$$

$$\Rightarrow 7^{2x+8} = 7^{3x+5}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow 2x+8=3x+5$$

$$\Rightarrow 3x-2x=8-5$$

$$\Rightarrow x = 3$$

Solution 2:

(i)

$$4^{2x} = \frac{1}{32}$$

$$\Rightarrow (2 \times 2)^{2x} = \frac{1}{2 \times 2 \times 2 \times 2 \times 2}$$

$$\Rightarrow (2^2)^{2x} = \frac{1}{2^5}$$

$$\Rightarrow 2^{2 \times 2x} = 2^{-5}$$

$$\Rightarrow 2^{4x} = 2^{-5}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow 4x = -5$$

$$\Rightarrow x = \frac{-5}{4}$$

(ii)

$$\sqrt{2^{x+3}} = 16$$

$$(2^{x+3})^{\frac{1}{2}} = 2 \times 2 \times 2 \times 2$$

$$\Rightarrow 2^{\frac{x+3}{2}} = 2^4$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x+3}{2} = 4$$

$$\Rightarrow x+3 = 8$$

$$\Rightarrow x = 8 - 3$$

$$\Rightarrow x = 5$$

(iii)

$$\left(\sqrt[3]{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$$

$$\Rightarrow \left[\left(\frac{3}{5}\right)^{\frac{1}{3}}\right]^{x+1} = \frac{5 \times 5 \times 5}{3 \times 3 \times 3}$$

$$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{3}} = \left(\frac{5}{3}\right)^3$$

$$\Rightarrow \left(\frac{3}{5}\right)^{\frac{x+1}{3}} = \left(\frac{3}{5}\right)^{-3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x+1}{3} = -3$$

$$\Rightarrow x+1 = -9$$

$$\Rightarrow x = -9 - 1$$

$$\Rightarrow x = -10$$

(iv)

$$\left(\sqrt[3]{\frac{2}{3}}\right)^{x-1} = \frac{27}{8}$$

$$\left[\left(\frac{2}{3}\right)^{\frac{1}{3}}\right]^{x-1} = \frac{3^3}{2^3}$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{3}{2}\right)^3$$

$$\Rightarrow \left(\frac{2}{3}\right)^{\frac{x-1}{3}} = \left(\frac{2}{3}\right)^{-3}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow \frac{x-1}{3} = -3$$

$$\Rightarrow x-1 = -9$$

$$\Rightarrow x = -9 + 1$$

$$\Rightarrow x = -8$$

Solution 3:

(i)

$$4^{x-2} - 2^{x+1} = 0$$

$$\Rightarrow 4^{x-2} = 2^{x+1}$$

$$\Rightarrow (2^2)^{x-2} = 2^{x+1}$$

$$\Rightarrow 2^{2x-4} = 2^{x+1}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow 2x - 4 = x + 1$$

$$\Rightarrow 2x - x = 4 + 1$$

$$\Rightarrow x = 5$$

(ii)

$$3^{x^2} : 3^x = 9 : 1$$

$$\frac{3^{x^2}}{3^x} = \frac{9}{1}$$

$$\Rightarrow 3^{x^2} = 9 \times 3^x$$

$$\Rightarrow 3^{x^2} = 3^2 \times 3^x$$

$$\Rightarrow 3^{x^2} = 3^{x+2}$$

We know that if bases are equal, the powers are equal

$$\Rightarrow x^2 = x + 2$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow x^2 - 2x + x - 2 = 0$$

$$\Rightarrow x(x - 2) + 1(x - 2) = 0$$

$$\Rightarrow (x + 1)(x - 2) = 0$$

$$\Rightarrow x + 1 = 0 \text{ or } x - 2 = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

Solution 4:

(i)

$$8 \times 2^{2x} + 4 \times 2^{x+1} = 1 + 2^x$$

$$\Rightarrow 8 \times (2^x)^2 + 4 \times 2^x \times 2^1 = 1 + 2^x$$

$$\Rightarrow 8 \times (2^x)^2 + 4 \times (2^x) \times 2^1 - 1 - 2^x = 0$$

$$\Rightarrow 8 \times (2^x)^2 + (2^x) \times (8 - 1) - 1 = 0$$

$$\Rightarrow 8 \times (2^x)^2 + 7(2^x) - 1 = 0$$

$$\Rightarrow 8y^2 + 7y - 1 = 0 \quad [y = 2^x]$$

$$\Rightarrow 8y^2 + 8y - y - 1 = 0$$

$$\Rightarrow 8y(y + 1) - 1(y + 1) = 0$$

$$\Rightarrow (8y - 1)(y + 1) = 0$$

$$\Rightarrow 8y = 1 \text{ or } y = -1$$

$$\Rightarrow y = \frac{1}{8} \text{ or } y = -1$$

$$\Rightarrow 2^x = \frac{1}{8} \text{ or } 2^x = -1$$

$$\Rightarrow 2^x = \frac{1}{2^3} \text{ or } 2^x = -1$$

$$\Rightarrow 2^x = 2^{-3} \text{ or } 2^x = -1$$

$$\Rightarrow x = -3$$

[$\because 2^x = -1$ is not possible]

(ii)

$$2^{2x} + 2^{x+2} - 4 \times 2^3 = 0$$

$$\Rightarrow (2^x)^2 + 2^x \cdot 2^2 - 4 \times 2 \times 2 \times 2 = 0$$

$$\Rightarrow (2^x)^2 + 2^x \cdot 2^2 - 32 = 0$$

$$\Rightarrow y^2 + 4y - 32 = 0 \quad [y = 2^x]$$

$$\Rightarrow y^2 + 8y - 4y - 32 = 0$$

$$\Rightarrow y(y+8) - 4(y+8) = 0$$

$$\Rightarrow (y+8)(y-4) = 0$$

$$\Rightarrow y+8 = 0 \text{ or } y-4 = 0$$

$$\Rightarrow y = -8 \text{ or } y = 4$$

$$\Rightarrow 2^x = -8 \text{ or } 2^x = 4$$

$$\Rightarrow 2^x = 2^2 \quad [\because 2^x = -8 \text{ is not possible}]$$

$$\Rightarrow x = 2$$

(iii)

$$(\sqrt{3})^{x-3} = (\sqrt[4]{3})^{x+1}$$

$$\Rightarrow \left(3^{\frac{1}{2}}\right)^{x-3} = \left(3^{\frac{1}{4}}\right)^{x+1}$$

$$\Rightarrow 3^{\frac{x-3}{2}} = 3^{\frac{x+1}{4}}$$

$$\Rightarrow \frac{x-3}{2} = \frac{x+1}{4}$$

$$\Rightarrow 4(x-3) = 2(x+1)$$

$$\Rightarrow 4x - 12 = 2x + 2$$

$$\Rightarrow 4x - 2x = 12 + 2$$

$$\Rightarrow 2x = 14$$

$$\Rightarrow x = \frac{14}{2}$$

$$\Rightarrow x = 7$$

Solution 5:

$$4^{2m} = \left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow 4^{2m} = \left(\sqrt{8}\right)^2 \dots (1)$$

and

$$\left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2 \dots (2)$$

From (1)

$$4^{2m} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow \left(2^2\right)^{2m} = \left(\sqrt{2^3}\right)^2$$

$$\Rightarrow 2^{4m} = \left[\left(2^3\right)^{\frac{1}{2}}\right]^2$$

$$\Rightarrow 2^{4m} = \left[2^{3 \times \frac{1}{2}}\right]^2$$

$$\Rightarrow 2^{4m} = 2^{3 \times \frac{1}{2} \times 2}$$

$$\Rightarrow 2^{4m} = 2^3$$

$$\Rightarrow 4m = 3$$

$$\Rightarrow m = \frac{3}{4}$$

From (2), we have

$$\left(\sqrt[3]{16}\right)^{-\frac{6}{n}} = \left(\sqrt{8}\right)^2$$

$$\Rightarrow \left(\sqrt[3]{2 \times 2 \times 2 \times 2}\right)^{-\frac{6}{n}} = \left(\sqrt{2 \times 2 \times 2}\right)^2$$

$$\Rightarrow \left(\sqrt[3]{2^4}\right)^{-\frac{6}{n}} = \left(\sqrt{2^3}\right)^2$$

$$\Rightarrow \left[\left(2^4\right)^{\frac{1}{3}}\right]^{-\frac{6}{n}} = \left[\left(2^3\right)^{\frac{1}{2}}\right]^2$$

$$\Rightarrow \left[2^{\frac{4}{3}}\right]^{-\frac{6}{n}} = \left[2^{\frac{3}{2}}\right]^2$$

$$\Rightarrow 2^{\frac{4}{3} \times \left(-\frac{6}{n}\right)} = 2^{\frac{3}{2} \times 2}$$

$$\Rightarrow 2^{\left(-\frac{8}{n}\right)} = 2^3$$

$$\Rightarrow -\frac{8}{n} = 3$$

$$\Rightarrow n = \frac{-8}{3} \quad \text{Thus } m = \frac{3}{4} \quad n = \frac{-8}{3}$$

Solution 6:

Consider the equation

$$(\sqrt{32})^x + 2^{y+1} = 1$$

$$\Rightarrow (\sqrt{2 \times 2 \times 2 \times 2 \times 2})^x + 2^{y+1} = 1$$

$$\Rightarrow (\sqrt{2^5})^x + 2^{y+1} = 1$$

$$\Rightarrow \left[(2^5)^{\frac{1}{2}} \right]^x + 2^{y+1} = 2^0$$

$$\Rightarrow 2^{\frac{5x}{2}} + 2^{y+1} = 2^0$$

$$\Rightarrow \frac{5x}{2} - (y+1) = 0$$

$$\Rightarrow 5x - 2(y+1) = 0$$

$$\Rightarrow 5x - 2y - 2 = 0 \dots (1)$$

Now consider the other equation

$$8^y - 16^{4-\frac{x}{2}} = 0$$

$$\Rightarrow (2^3)^y - (2^4)^{4-\frac{x}{2}} = 0$$

$$\Rightarrow 2^{3y} - 2^{4(4-\frac{x}{2})} = 0$$

$$\Rightarrow 2^{3y} = 2^{4(4-\frac{x}{2})}$$

$$\Rightarrow 3y = 4\left(4 - \frac{x}{2}\right)$$

$$\Rightarrow 3y = 16 - 2x$$

$$\Rightarrow 2x + 3y = 16 \dots (2)$$

Thus we have two equations,

$$5x - 2y = 2 \dots\dots(1)$$

$$2x + 3y = 16\dots\dots(2)$$

Multiplying (1) by 3 and (2) by 2, we have

$$15x - 6y = 6\dots\dots(3)$$

$$4x + 6y = 32\dots\dots(4)$$

Adding (3) and (4), we have

$$19x = 38$$

$$\Rightarrow x = 2$$

Substituting the value of x in equation (1), we have,

$$5(2) - 2y = 2$$

$$\Rightarrow 10 - 2y = 2$$

$$\Rightarrow 2y = 10 - 2$$

$$\Rightarrow 2y = 8$$

$$\Rightarrow y = \frac{8}{2}$$

$$\Rightarrow y = 4$$

Thus the values of x and y are:

$$x = 2 \text{ and } y = 4$$

Solution 7:

(i)

$$\begin{aligned}\text{L.H.S.} &= \left(\frac{x^a}{x^b}\right)^{a+b-c} \times \left(\frac{x^b}{x^c}\right)^{b+c-a} \times \left(\frac{x^c}{x^a}\right)^{c+a-b} \\&= \left(x^{a-b}\right)^{(a+b-c)} \times \left(x^{b-c}\right)^{(b+c-a)} \times \left(x^{c-a}\right)^{(c+a-b)} \\&= x^{(a-b)(a+b-c)} \times x^{(b-c)(b+c-a)} \times x^{(c-a)(c+a-b)} \\&= x^{a^2+ab-ac-ab-b^2+bc} \times x^{b^2+bc-ab-cb-c^2+ac} \times x^{c^2+ac-bc-ac-a^2+ab} \\&= x^{a^2-ac-b^2+bc+b^2-ab-c^2+ac+c^2-bc-a^2+ab} \\&= x^0 \\&= 1 \\&= \text{R.H.S.}\end{aligned}$$

(ii)

We need to prove that

$$\frac{x^{a(b-c)}}{x^{b(a-c)}} \div \left(\frac{x^b}{x^a}\right)^c = 1$$

$$\begin{aligned}\text{L.H.S.} &= x^{a(b-c)-b(a-c)} \div \frac{x^{bc}}{x^{ac}} \\&\Rightarrow = x^{ab-ac-ab+bc} \div x^{bc-ac} \\&\Rightarrow = x^{ab-ac-ab+bc-(bc-ac)} \\&\Rightarrow = x^{ab-ac-ab+bc-bc+ac} \\&\Rightarrow = x^0 \\&\Rightarrow = 1 \\&\Rightarrow = \text{R.H.S.}\end{aligned}$$

Solution 8:

We are given that

$$a^x = b, b^y = c \text{ and } c^z = a$$

Consider the equation

$$a^x = b$$

$$\Rightarrow a^{xyz} = b^{yz} \quad [\text{raising to the power } yz \text{ on both sides}]$$

$$\Rightarrow a^{xyz} = (b^y)^z$$

$$\Rightarrow a^{xyz} = (c)^z \quad [\because b^y = c]$$

$$\Rightarrow a^{xyz} = c^z$$

$$\Rightarrow a^{xyz} = a \quad [\because c^z = a]$$

$$\Rightarrow a^{xyz} = a^1$$

$$\Rightarrow xyz = 1$$

Solution 9:

$$\text{Let } a^x = b^y = c^z = k$$

$$\therefore a = k^{\frac{1}{x}}; b = k^{\frac{1}{y}}; c = k^{\frac{1}{z}}$$

Also, we have $b^2 = ac$

$$\therefore \left(k^{\frac{1}{y}}\right)^2 = \left(k^{\frac{1}{x}}\right) \times \left(k^{\frac{1}{z}}\right)$$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{1}{x} + \frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}} = k^{\frac{z+x}{xz}}$$

Comparing the powers we have

$$\frac{2}{y} = \frac{z+x}{xz}$$

$$\Rightarrow y = \frac{2xz}{z+x}$$

Solution 10:

$$\text{Let } 5^{-p} = 4^{-q} = 20^r = k$$

$$5^{-p} = k \Rightarrow 5 = k^{\frac{-1}{p}} [\because a^p = b^q \Rightarrow a = b^{\frac{q}{p}}]$$

$$4^{-q} = k \Rightarrow 4 = k^{\frac{-1}{q}} [\because a^p = b^q \Rightarrow a = b^{\frac{q}{p}}]$$

$$20^r = k \Rightarrow 20 = k^{\frac{1}{r}} [\because a^p = b^q \Rightarrow a = b^{\frac{q}{p}}]$$

$$5 \times 4 = 20$$

$$\Rightarrow k^{\frac{-1}{p}} \times k^{\frac{-1}{q}} = k^{\frac{1}{r}}$$

$$\Rightarrow k^{\frac{-1}{p} - \frac{1}{q}} = k^{\frac{1}{r}}$$

$$\Rightarrow k^0 = k^{\frac{1}{p} + \frac{1}{q} + \frac{1}{r}}$$

$$\Rightarrow \frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 0 \text{ [If bases are equal, powers are also equal]}$$

Solution 11:

$$(m+n)^{-1}(m^{-1}+n^{-1}) = m^x n^y$$

$$\Rightarrow \frac{1}{(m+n)} \times \left(\frac{1}{m} + \frac{1}{n} \right) = m^x n^y$$

$$\Rightarrow \frac{1}{(m+n)} \times \left(\frac{m+n}{mn} \right) = m^x n^y$$

$$\Rightarrow \frac{1}{mn} = m^x n^y$$

$$\Rightarrow m^{-1}n^{-1} = m^x n^y$$

Comparing the coefficients of x and y , we get

$$x = -1 \text{ and } y = -1$$

LHS,

$$x + y + 2 = (-1) + (-1) + 2 = 0 = \text{RHS}$$

Solution 12:

$$5^{x+1} = 25^{x-2}$$

$$\Rightarrow 5^{x+1} = (5^2)^{x-2}$$

$$\Rightarrow 5^{x+1} = 5^{2x-4} \text{ [If bases are equal, powers are also equal]}$$

$$\Rightarrow x + 1 = 2x - 4$$

$$\Rightarrow 2x - x = 4 + 1$$

$$\Rightarrow x = 5$$

$$\therefore 3^{x-3} \times 2^{3-x} = 3^{5-3} \times 2^{3-5} = 3^2 \times 2^{-2} = 9 \times \frac{1}{4} = \frac{9}{4}$$

Solution 13:

$$4^{x+3} = 112 + 8 \times 4^x$$

$$\Rightarrow 4^x \times 4^3 = 112 + 8 \times 4^x$$

$$\Rightarrow 64 \times 4^x = 112 + 8 \times 4^x$$

$$\text{Let } 4^x = y$$

$$64y = 112 + 8y$$

$$\Rightarrow 56y = 112$$

$$\Rightarrow y = 2$$

Substituting we get,

$$4^x = 2$$

$$\Rightarrow 2^{2x} = 2$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$$(18x)^{3x} = \left(\frac{18}{2}\right)^{3 \times \frac{1}{2}} = 9^{3 \times \frac{1}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$$

Solution 14(i):

(i)

$$\begin{aligned}
4^{x-1} \times (0.5)^{3-2x} &= \left(\frac{1}{8}\right)^{-x} \\
\Rightarrow (2^2)^{x-1} \times \left(\frac{1}{2}\right)^{3-2x} &= \left(\frac{1}{2^3}\right)^{-x} \\
\Rightarrow 2^{2x-2} \times 2^{-(3-2x)} &= (2^{-3})^{-x} \\
\Rightarrow 2^{2x-2-3+2x} &= 2^{3x} \\
\Rightarrow 2^{4x-5} &= 2^{3x} \\
\Rightarrow 4x - 5 &= 3x \\
\Rightarrow 4x - 3x &= 5 \\
\Rightarrow x &= 5
\end{aligned}$$

Solution 14(ii):

$$\begin{aligned}
a^{2(3x+5)} \times a^{4x} &= a^{8x+12} \\
\Rightarrow a^{6x+10+4x} &= a^{8x+12} \\
\Rightarrow 10x + 10 &= 8x + 12 \text{ [If bases are the same, powers are also same]} \\
\Rightarrow 2x &= 2 \\
\Rightarrow x &= 1
\end{aligned}$$

Solution 14(iii):

$$\begin{aligned}
(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + x \left(\frac{1}{2}\right)^{-1} \cdot 2^0 &= 27 \\
\Rightarrow 3^{4 \times \frac{3}{4}} - (2^{-5})^{-\frac{2}{5}} + x(2) &= 27 \\
\Rightarrow 3^3 - 2^2 + 2x &= 27 \\
\Rightarrow 2x + 27 - 4 &= 27 \\
\Rightarrow 2x &= 4 \\
\Rightarrow x &= 2
\end{aligned}$$

Solution 14(iv):

$$\begin{aligned}2^{3x} \times 2^3 &= 2^{3x} \times 2 + 48 \\ \Rightarrow 8 \times 2^{3x} &= 2^{3x} \times 2 + 48 \\ \Rightarrow 2^{3x} (8 - 2) &= 48 \\ \Rightarrow 2^{3x} \times 6 &= 48 \\ \Rightarrow 2^{3x} &= 8 \\ \Rightarrow 2^{3x} &= 2^3 \\ \Rightarrow 3x &= 3 \\ \Rightarrow x &= 1\end{aligned}$$

Solution 14(v):

$$\begin{aligned}3 \times 2^x + 3 - 2^x \times 2^2 + 5 &= 0 \\ \Rightarrow 2^x (3 - 4) + 8 &= 0 \\ \Rightarrow -2^x &= -8 \\ \Rightarrow 2^x &= 8 \\ x &= 3\end{aligned}$$

Exercise 7(C)

Solution 1:

$$\begin{aligned}\text{(i)} \quad & 9^{\frac{5}{2}} - 3 \times 8^{\circ} - \left(\frac{1}{81}\right)^{-\frac{1}{2}} \\ &= (3^2)^{\frac{5}{2}} - 3 \times 1 - \left(\frac{1}{3^4}\right)^{-\frac{1}{2}} \\ &= 3^{2 \times \frac{5}{2}} - 3 - 3^{-4 \times \left(-\frac{1}{2}\right)} \\ &= 3^5 - 3 - 3^2 \\ &= 243 - 3 - 9 \\ &= 231\end{aligned}$$

$$\begin{aligned}\text{(ii)} \quad & (64)^{\frac{2}{3}} - \sqrt[3]{125} - \frac{1}{2^{-5}} + (27)^{-\frac{2}{3}} \times \left(\frac{25}{9}\right)^{-\frac{1}{2}} \\ &= (4^3)^{\frac{2}{3}} - \sqrt[3]{5^3} - 2^5 + (3^3)^{-\frac{2}{3}} \times \left(\frac{5^2}{3^2}\right)^{-\frac{1}{2}} \\ &= 4^2 - 5 - 2^5 + 3^{-2} \times \left(\frac{5}{3}\right)^{2 \times \left(-\frac{1}{2}\right)} \\ &= 16 - 5 - 32 + \frac{1}{3^2} \times \left(\frac{5}{3}\right)^{-1} \\ &= -21 + \frac{1}{9} \times \frac{3}{5} \\ &= -21 + \frac{1}{15} \\ &= \frac{-315 + 1}{15} \\ &= \frac{-314}{15} \\ &= -20\frac{14}{15}\end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad & \left[\left(-\frac{2}{3} \right)^{-2} \right]^3 \times \left(\frac{1}{3} \right)^{-4} \times 3^{-1} \times \frac{1}{6} \\
 &= \left[\left(-\frac{3}{2} \right)^2 \right]^3 \times (3)^4 \times \frac{1}{3} \times \frac{1}{3 \times 2} \\
 &= \left(-\frac{3}{2} \right)^6 \times (3)^2 \times \frac{1}{2} \\
 &= \frac{3^{6+2}}{2^{6+1}} \\
 &= \frac{3^8}{2^7}
 \end{aligned}$$

Solution 2:

$$\begin{aligned}
 & \frac{3 \times 9^{n+1} - 9 \times 3^{2n}}{3 \times 3^{2n+3} - 9^{n+1}} \\
 &= \frac{3 \times (3^2)^{n+1} - 3^2 \times 3^{2n}}{3 \times 3^{2n+3} - (3^2)^{n+1}} \\
 &= \frac{3^{1+2n+2} - 3^{2+2n}}{3^{1+2n+3} - 3^{2n+2}} \\
 &= \frac{3^{3+2n} - 3^{2+2n}}{3^{4+2n} - 3^{2n+2}} \\
 &= \frac{3^{2n}(3^3 - 3^2)}{3^{2n}(3^4 - 3^2)} \\
 &= \frac{27 - 9}{81 - 9} \\
 &= \frac{18}{72} \\
 &= \frac{1}{4}
 \end{aligned}$$

Solution 3:

$$3^{x-1} \times 5^{2y-3} = 225$$

$$\Rightarrow 3^{x-1} \times 5^{2y-3} = 3^2 \times 5^2$$

$$\Rightarrow x - 1 = 2 \text{ and } 2y - 3 = 2$$

$$\Rightarrow x = 3 \text{ and } 2y = 5$$

$$\Rightarrow x = 3 \text{ and } y = \frac{5}{2}$$

$$\Rightarrow x = 3 \text{ and } y = 2\frac{1}{2}$$

Solution 4:

$$\left(\frac{a^{-1}b^2}{a^2b^{-4}}\right)^7 \div \left(\frac{a^3b^{-5}}{a^{-2}b^3}\right)^{-5} = a^x \cdot b^y$$

$$\Rightarrow \left(\frac{b^6}{a^3}\right)^7 \div \left(\frac{a^5}{b^8}\right)^{-5} = a^x \cdot b^y$$

$$\Rightarrow \left(\frac{b^6}{a^3}\right)^7 \div \left(\frac{b^8}{a^5}\right)^5 = a^x \cdot b^y$$

$$\Rightarrow \frac{b^{42}}{a^{21}} \div \frac{b^{40}}{a^{25}} = a^x \cdot b^y$$

$$\Rightarrow \frac{b^{42}}{a^{21}} \times \frac{a^{25}}{b^{40}} = a^x \cdot b^y$$

$$\Rightarrow b^2 \times a^4 = a^x \times b^y$$

$$\Rightarrow x = 4 \text{ and } y = 2$$

$$\Rightarrow x + y = 4 + 2 = 6$$

Solution 5:

$$3^{x+1} = 9^{x-3}$$

$$\Rightarrow 3^x \times 3 = (3^2)^{x-3}$$

$$\Rightarrow 3^x \times 3 = 3^{2x-6}$$

$$\Rightarrow 3^x \times 3 = \frac{3^{2x}}{3^6}$$

$$\Rightarrow 3^6 \times 3 = \frac{3^{2x}}{3^x}$$

$$\Rightarrow 3^7 = 3^x$$

$$\Rightarrow x = 7$$

$$\Rightarrow 2^{1+x} = 2^{1+7} = 2^8 = 256$$

Solution 6:

$$2^x = 4^y = 8^z$$

$$\Rightarrow 2^x = 2^{2y} = 2^{3z}$$

$$\Rightarrow x = 2y = 3z$$

$$\Rightarrow y = \frac{x}{2} \text{ and } z = \frac{x}{3}$$

$$\text{Now, } \frac{1}{2x} + \frac{1}{4y} + \frac{1}{8z} = 4$$

$$\Rightarrow \frac{1}{2x} + \frac{1}{\frac{4x}{2}} + \frac{1}{\frac{8x}{3}} = 4$$

$$\Rightarrow \frac{1}{2x} + \frac{2}{4x} + \frac{3}{8x} = 4$$

$$\Rightarrow \frac{1}{2x} + \frac{1}{2x} + \frac{3}{8x} = 4$$

$$\Rightarrow \frac{4+4+3}{8x} = 4$$

$$\Rightarrow \frac{11}{8x} = 4$$

$$\Rightarrow x = \frac{11}{32}$$

Solution 7:

$$\frac{9^n \cdot 3^2 \cdot 3^n - (27)^n}{(3^m \cdot 2)^3} = 3^{-3}$$

$$\Rightarrow \frac{3^{2n} \cdot 3^2 \cdot 3^n - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \cdot 3^2 - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \times 8}{3^{3m} \times 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{1}{3^{3(m-n)}} = \frac{1}{3^{3 \times 1}}$$

$$\Rightarrow m - n = 1 \quad (\text{proved})$$

Solution 8:

$$(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

$$\Rightarrow (13)^{\sqrt{x}} = 256 - 81 - 6$$

$$\Rightarrow (13)^{\sqrt{x}} = 169$$

$$\Rightarrow (13)^{\sqrt{x}} = 13^2$$

$$\Rightarrow \sqrt{x} = 2$$

$$\Rightarrow x = 4$$

Solution 9:

$$3^{4x} = (81)^{-1} \text{ and } (10)^{\frac{1}{y}} = 0.0001$$

$$\Rightarrow 3^{4x} = (3^4)^{-1} \text{ and } (10)^{\frac{1}{y}} = \frac{1}{10000}$$

$$\Rightarrow 3^{4x} = 3^{-4} \text{ and } (10)^{\frac{1}{y}} = \frac{1}{10^4}$$

$$\Rightarrow 4x = -4 \text{ and } (10)^{\frac{1}{y}} = 10^{-4}$$

$$\Rightarrow x = -1 \text{ and } \frac{1}{y} = -4$$

$$\Rightarrow x = -1 \text{ and } y = -\frac{1}{4}$$

$$\therefore 2^{-x} \times 16^y = 2^{-(-1)} \times 16^{-\frac{1}{4}}$$

$$= 2 \times 2^{4 \times \left(-\frac{1}{4}\right)}$$

$$= 2 \times 2^{-1}$$

$$= 2^{1-1}$$

$$= 2^0$$

$$= 1$$

Solution 10:

$$\begin{aligned}
3(2^x + 1) - 2^{x+2} + 5 &= 0 \\
\Rightarrow 3 \times 2^x + 3 - 2^x \times 2^2 + 5 &= 0 \\
\Rightarrow 2^x(3 - 2^2) + 8 &= 0 \\
\Rightarrow 2^x(3 - 4) &= -8 \\
\Rightarrow 2^x \times (-1) &= -8 \\
\Rightarrow 2^x &= 8 \\
\Rightarrow 2^x &= 2^3 \\
\Rightarrow x &= 3
\end{aligned}$$

Solution 11:

$$\begin{aligned}
(a^m)^n &= a^m \cdot a^n \\
\Rightarrow a^{mn} &= a^{m+n} \\
\Rightarrow mn &= m + n \quad \dots(1) \\
\text{Now,} \\
m(n-1) - (n-1) \\
&= mn - m - n + 1 \\
&= m + n - m - n + 1 \quad \dots [\text{From (1)}] \\
&= 1
\end{aligned}$$

Solution 12:

$$\begin{aligned}
m &= \sqrt[3]{15} \text{ and } n = \sqrt[3]{14} \\
\Rightarrow m^3 &= 15 \text{ and } n^3 = 14 \\
\therefore m - n - \frac{1}{m^2 + mn + n^2} &= \frac{(m^3 + m^2n + mn^2) - (m^2n + mn^2 + n^3) - 1}{m^2 + mn + n^2} \\
&= \frac{m^3 + m^2n + mn^2 - m^2n - mn^2 - n^3 - 1}{m^2 + mn + n^2} \\
&= \frac{m^3 - n^3 - 1}{m^2 + mn + n^2} \\
&= \frac{15 - 14 - 1}{m^2 + mn + n^2} \\
&= \frac{1 - 1}{m^2 + mn + n^2} \\
&= 0
\end{aligned}$$